$$\therefore r^2=\frac{2a(A\cos^2\theta+B\sin^2\theta)-1}{(A\cos^2\theta+B\sin^2\theta)^2}=R^2.$$

$$\therefore v=\frac{4a^3}{3}\int_0^{\frac{1}{2}\pi}\left[1-\left(\frac{A\cos^2\theta+B\sin^2\theta-1}{A\cos^2\theta+B\sin^2\theta}\right)^3\right]d\theta$$

$$=\frac{4a^3}{3}\int_0^{\frac{1}{2}\pi}\left[\frac{3}{A\cos^2\theta+B\sin^2\theta}-\frac{3}{(A\cos^2\theta+B\sin^2\theta)^2}+\frac{1}{(A\cos^2\theta+B\sin^2\theta)^3}\right]d\theta$$

$$=\frac{2\pi a^3}{\sqrt{AB}}\left[1-\frac{A+B}{2AB}+\frac{A^2+B^2}{8A^2B^2}\right]+\frac{\pi a^3}{6\sqrt{\{A^3B^3\}}}.$$

(*b*). From (1) and (3), $R=\dfrac{2a\sqrt{\{A\cos^2\theta+B\sin^2\theta\}}}{1+A\cos^2\theta+B\sin^2\theta}=\dfrac{2aC}{D}$.

$$\therefore v=\frac{4a^3}{3}\int_0^{\frac{1}{2}\pi}\left[1-\left(\frac{C^2-1}{D}\right)^3\right]d\theta=\frac{8a^3}{3}\int_0^{\frac{1}{2}\pi}\left(\frac{3}{D}-\frac{6}{D^2}+\frac{4}{D^3}\right)d\theta$$

$$=\frac{2\pi a^3}{\sqrt{\{(A+1)(B+1)\}}}\left[\left(\frac{A}{A+1}\right)^2+\left(\frac{B}{B+1}\right)^2\right]+\frac{4\pi a^3}{3\sqrt{\{(A+1\}^3\{B+1\}^3}}.$$

145. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the surface bounding the volume required in problem 102.

Solution by the PROPOSER.

The surface is composed of a portion of the surface of the cone, and a portion of the surface of the paraboloid.

The equation to the cone is $x^2+y^2=c^2z^2$.

The equation to the paraboloid is $y^2+z^2=4a(a+x)$.

$S=S_c+S_p$.

$S_c=\dfrac{2}{c}\displaystyle\int_{x_2}^{x_1}\int_0^{y_1}\sqrt{\{1+c^2\}}dxdy$, {see 102 for limits}

$$=\frac{2}{c}\int_{x_2}^{x_1}\sqrt{\{4ac^2(a+x)-x^2\}}dx.$$

Let $x=2ac^2-2ac\sqrt{\{1+c^2\}}\cos\theta$.

$$\therefore S_c=8a^2c(1+c^2)\int_0^{\pi}\sin^2\theta d\theta=4\pi a^2c(1+c^2).$$

$$S_p=4\int_{x_2}^{x_1}\int_0^{y_1}\sqrt{\frac{a(2a+x)}{4a(a+x)-y^2}}dxdy=4\int_{x_2}^{x_1}\sqrt{\{a(2a+x)\}}\sin^{-1}\sqrt{\frac{4ac^2(a+x)-x^2}{4a(1+c^2)(a+x)}}dx$$

$$=\frac{4}{3a}\int_{x_2}^{x_1}\frac{x(2a^2+ax)^{\frac{3}{2}}}{(a+x)\sqrt{\{4ac^2(a+x)-x^2\}}}dx$$

$$=\frac{4a}{3}\int_{x_2}^{x_1}\frac{x\sqrt{\{2a^2+ax\}}dx}{(a+x)\sqrt{\{4ac^2(a+x)-x^2\}}}+\tfrac{4}{3}\int_{x_2}^{x_1}\frac{x\sqrt{\{2a^2+ax\}}dx}{\sqrt{\{4ac^2(a+x)-x^2\}}}.$$

Let $x=2ac^2+2ac\sqrt{\{1+c^2\}}\cos2\theta$, $1+c^2+c\sqrt{\{1+c^2\}}=b^2$, $2c\sqrt{\{1+c^2\}}/b^2=e^2$, $4c\sqrt{\{1+c^2\}}/\{2b^2-1\}=d$.

$$\therefore\ S_p=\frac{16a^2b\sqrt{2(b^2-1)}}{3(2b^2-1)}\int_0^{\frac{1}{2}\pi}\frac{d\theta}{(1-d\sin^2\theta)\sqrt{\{1-e^2\sin^2\theta\}}}$$

$$-\frac{16a^2b\sqrt{2(2c\sqrt{\{1+c^2\}}+b^2e^2-e^2)}}{3(2b^2-1)}\int_0^{\frac{1}{2}\pi}\frac{\sin^2\theta d\theta}{(1-d\sin^2\theta)\sqrt{\{1-e^2\sin^2\theta\}}}$$

$$+\frac{32a^2bce^2\sqrt{\{2(1+c^2)\}}}{3(2b^2-1)}\int_0^{\frac{1}{2}\pi}\frac{\sin^4\theta d\theta}{(1-d\sin^2\theta)\sqrt{\{1-e^2\sin^2\theta\}}}$$

$$+\tfrac{16}{3}a^2b\sqrt{2(b^2-1)}\int_0^{\frac{1}{2}\pi}\sqrt{\{1-e^2\sin^2\theta\}}d\theta$$

$$-\tfrac{32}{3}a^2bc\sqrt{\{2(1+c^2)\}}\int_0^{\frac{1}{2}\pi}\sqrt{\{1-e^2\sin^2\theta\}}\sin^2\theta d\theta.$$

$$\therefore\ S_p=A\Pi(e,\ -d,\ \tfrac{1}{2}\pi)+\frac{B}{d}\{F(e,\ \tfrac{1}{2}\pi)-\Pi(e,\ -d,\ \tfrac{1}{2}\pi)\}$$

$$+\frac{C}{d^2e^2}\{e^2\Pi(e,\ -d,\ \tfrac{1}{2}\pi)+dE(e,\ \tfrac{1}{2}\pi)-(d+e^2)F(e,\ \tfrac{1}{2}\pi)\}+DE(e,\ \tfrac{1}{2}\pi)$$

$$+\frac{E}{3e^2}\{1-2e^2)E(e,\ \tfrac{1}{2}\pi)-(1-e^2)F(e,\ \tfrac{1}{2}\pi)\}.$$

$$\therefore\ S=4\pi a^2c(1+c^2)+(A-\frac{B}{d}+\frac{C}{d^2})\Pi(e,\ -d,\ \tfrac{1}{2}\pi)+\left(\frac{C}{de^2}+D\right.$$

$$\left.+\frac{E(1-2e^2)}{3e^2}\right)E(e,\ \tfrac{1}{2}\pi)+\left(\frac{B}{d}-\frac{C(d+e^2)}{d^2e^2}-\frac{E(1-e^2)}{3e^2}\right)F(e,\ \tfrac{1}{2}\pi).$$

MECHANICS.

139. **Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College. Defiance, Ohio.**

A homogeneous sphere, radius $r=50$ inches, makes $m=30$ revolutions around an axis every second. The mass begins to disappear from the surface into space at a rate exactly sufficient to cause the diameter to decrease uniformly at the rate of $(1/n)$th$=1/1000$th of